"Fundamentals and Optimal Institutions: The case of US sports leagues"

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Fundamentals and Optimal Institutions: The case of US sports leagues*

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Abstract

To shed light on the relation between fundamentals and adopted institutions we examine institutional choice across the "Big Four" US sports leagues. Despite having very similar business models and facing the same economic and legal environment, these leagues exhibit large differences in their use of regulatory institutions such as revenue sharing, salary caps or luxury taxes. We show, theoretically and

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empirically, that these institutional differences can be rationalized as optimal responses to differences in the fundamental characteristics of the sports being played. This provides a cautionary tale against trying to transplant successful institutions across different economic settings.

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1 Introduction

Whether comparing countries, firms or other economic entities, it is well established that institutions typically vary across different settings. The reason for these differences is less clear. On the one hand, institutional differences may simply arise through a combination of random chance and path dependence. On the other hand, it may be that the observed differences in institutions across settings reflect optimal responses to differences in the underlying preferences, technology or other fundamentals. Distinguishing between these possible explanations is of first order importance for providing policy prescriptions. In particular, if differences in institutions are largely caused by random chance, we can expect large gains from identifying and transplanting successful institutions from one context to another. Understanding the relationship between fundamentals and institutions is a challenging task, however, as institutions are complex objects and fundamentals often differ in a myriad of ways across economic settings.

To study the interplay between fundamentals and adopted institutions, this paper analyzes the specific case of institutional choice across the "Big Four" US sports leagues: the National Football League (NFL), the National Basketball Association (NBA), the National Hockey League (NHL), and Major League Baseball (MLB).[1] Each of these leagues currently consists of about 30 teams that make hiring decisions and play games against each other to generate revenue from fans and media contracts. To regulate this process

[1]Except for the NFL, all these leagues have at least one team in a Canadian city so a more precise description might be "North American" sports. For brevity we refer to them as US sports however.

each league decides on a set of institutions. In particular the league chooses the extent to which it will regulate teams' hiring of players, either through the use of direct constraints such as a salary cap, or through redistributive policies such as revenue sharing or various forms of payroll taxes.

For studying the relationship between institutions and fundamentals, these US sports leagues are particularly well suited as a case study since they face very similar fundamental conditions regarding revenue generation. They all draw most of their fans from the US population, negotiate broadcasting contracts with the same networks, and operate under the same legal system.[2] At the same time, the four US sports leagues exhibit large differences in their choice of institutions. At one end, the NFL features both a hard cap on salaries and extensive revenue sharing across teams. At the other end, the MLB has significantly less revenue sharing and only a modest payroll tax. These differences in institutions have periodically led league officials and commentators to argue that institutions should be transplanted across leagues.[3] Importantly, the four leagues differ in terms of one fundamentally basic premise: the rules of the sports that they play. In this paper we show that the observed differences in institutions across sports leagues can be rationalized as reflecting optimal responses of the leagues to differences in the fundamental rules of the underlying sports.

Our analysis starts from a standard off-the-shelf league model from the sports literature. In the model, each team in the league invests in a one-dimensional input, called skill or talent, which increases its chance of winning a given game, and thus the championship. Given this investment, the team will attract fans and viewers, from whom it will collect revenue. Relative to the standard model, we introduce two simple modifications. First, to allow leagues' institutions to influence the total amount of skills hired by teams, we do away with the assumption of a fixed supply of talent. Second, we allow for heterogeneity in the rules of the game reflected in one parameter, the elasticity of the odds of winning a

[2]An outlier is the NHL which has seven teams in Canadian cities (NBA and MLB each have only one Canadian team).

[3]As an example of the former see Levin et al. (2000). Examples of the latter are Rogers (2015) and Gordon (2012).

match with respect to the relative skills of the playing teams. We denote this parameter the *productivity of skills*.

We analyze teams' individual hiring decisions and compare the allocation of talent to the choice made by a planner that maximizes total league profits. Externalities arise because each team's talent level affects the revenue and win probabilities of other teams. As a result teams will in general not hire the efficient amount of skills.[4] These externalities thus provide a rationale for leagues to introduce regulatory and redistributive institutions. When teams have an incentive to hire more skills than the efficient amount, revenue sharing can increase aggregate profits by depressing teams' hiring incentives. Moreover, the strength of these externalities depends on the productivity of skills, as the incentives to hire increase when skills have a stronger effect on win probability. As a result, it is optimal for leagues that play sports with a higher productivity of skills to introduce a higher level of revenue sharing and other regulatory institutions that affect hiring incentives.

We use recent data on match outcomes and team payrolls to provide estimates of the productivity of skills across the four major US sports leagues. We find large differences across the four leagues. The elasticity of the odds of winning a match with respect to the relative skills ranges from about 1.54 in the NBA, to about 0.16 in MLB. Comparing these estimated productivity of skills with the institutions actually observed in the different leagues, we find that the theoretical predictions from our theoretical model fit the data well. For three of the four leagues, NBA, NHL and MLB, their rankings in terms of regulation and productivity of skills exactly mirror the predictions from the model. The NBA is the most heavily regulated and also has the highest estimated productivity of skills, while MLB is the least regulated and has the lowest estimated productivity of skills. Moreover the differences in the productivity of skills across these three leagues are all statistically significant. For the last league, the NFL, the estimated productivity of skills is lower than our model would predict. Due to the low number of games per season in the NFL, however, our estimate for the NFL is also very imprecise so we cannot reject that the NFL in fact has the highest productivity of skills. We conclude that our proposed

[4] We refer to efficiency with respect to the objective of maximization of league-wide profits.

explanation rationalizes the institutional differences across US sports leagues well. Sports where the productivity of skills is higher have more revenue sharing and other regulatory institutions relative to leagues from sports with lower productivity of skills.

Our paper makes several contributions to the existing literature on the economics of sports (for reviews of this literature see Downward and Dawson 2000; Andreff and Szymanski 2006; Downward, Dawson, and Dejonghe 2009). Relative to this literature, our model highlights the importance of a parameter, the productivity of skills, that has not been studied so far. Its main advantage is that, being related to match level data, it represents an intrinsic characteristic of a given sport that is unaffected by league institutions and regulations. In contrast, most existing comparisons across sports consider outcome measures that respond endogenously to institutions, such as the probability of winning the championship. To the best of our knowledge, we are also the first to explicitly model season length as another institution chosen by the league. In particular, we show that under certain conditions increasing season length increases the responsiveness of the probability of winning a championship to teams' skills. This in turn yields predictions about which sports leagues should have longer seasons that are in line with what we observe in the data.

At the broader level, our paper contributes to the literature on the optimal design of institutions by highlighting the crucial role of fundamentals in shaping institutions. In particular, our results provide a cautionary tale against the common practice of advocating the transplant of institutions across contexts.[5] Similar arguments have been made in the existing literature and debate. Much of the recent critique of the Washington Consensus can be viewed as arguing for more focus on differences in fundamentals across countries.[6]

[5]Attempts to transplant institutions across contexts is a common policy prescription. As examples of actual reform from above we have the dissemination of the English, French and German legal traditions through conquest, colonization or imitation in the 19^{th} century. Japan in 1945 imported many of US institutions, and in 2004 Dubai adopted common law for its International Finance Centre. Transplanting institutions was in large part the essence of the Washington Consensus and, more recently, suggestions that the US should adopt some Scandinavian institutions have surfaced several times in the 2016 presidential primaries.

[6]See for example Stiglitz (2008).

Easterly (2008) argues that when institutions express social norms, traditions, and beliefs, change is always gradual. Finally, Acemoglu et al. (forthcoming) uses a similar argument to criticize the idea of exporting Scandinavian institutions to other countries. Relative to this existing literature we provide a well identified theoretical model and empirical case study of how institutions respond to fundamentals.

The paper continues as follows. Section 2 describes the context and institutions across the four major US sports leagues. Section 3 develops the model, while Section 4 presents the data and reports the results. Section 5 concludes.

2 US sports leagues

Professional sports in North America are dominated by the so-called "Big Four" leagues: NFL, NBA, NHL and MLB. The first professional league, for baseball, was founded in 1875, and the last, for basketball, was founded in 1946. Average attendance (and yearly revenues), as of 2015, is 17,500 (3.7 billion dollars) in the NHL, 17,800 (5.2 billion dollars) in the NBA, 30,500 (9.5 billion dollars) in the MLB, and 68,200 (13 billion dollars) in the NFL. In North America, the sports market was worth $63.5 billion in 2015, and the sports industry contributes approximately 500,000 jobs.[7]

Each of the four sports leagues comprise a stipulated number of teams, also known as franchises. Currently, the NFL has 32 teams, while the NBA, NHL, and MLB have 30 teams each. Although franchises are corporate entities separated from their leagues, they operate only under league auspices. The formal structure of the leagues is the cooperative association of team owners. All strategic questions of league-wide relevance are decided by majority voting, and only franchise owners are allowed to vote (Downward and Dawson 2000). The Big Four leagues have franchises placed nationwide, and all leagues grant territorial exclusivity to their owners, precluding the addition of another team in the same area unless the current team's owners consent. All four major leagues have strict rules regarding who may own a team, and generally do not allow anyone to own a stake

[7] See Heitner (2015).

in more than one franchise, to prevent the perception of being in a conflict of interest.

Viewed as economic entities, the activities of the Big Four leagues is very similar. Once a year a season takes place in which the teams of a league play games against each other in the respective sports. Teams make decisions about player hiring and coaching, most of them in-between seasons, which influence the outcome of the games. At the end of the season the most successful team is crowned as the winner (champion) of the league.[8] This process generates revenue for teams. Major revenue sources are admissions and tickets (35% of revenue), television and broadcasting rights fees (30%), advertising, sponsorships, and endorsement fees (10%), and concessions and merchandise sales (5%).[9]

All leagues use institutions that regulate the behavior of teams in various regards. In this paper we focus on regulatory institutions that affect teams' incentives to hire players, either through redistribution or direct constraints on team behavior. The simplest example of such a regulatory institution is "revenue sharing", which is essentially the sports league version of redistributive taxation. Assuming (reasonably) that hiring better players increases team revenue, revenue sharing depresses teams' hiring incentives.

The Big Four leagues all use revenue sharing, though they do so to very different degrees. In the NFL teams share close to 61% of all league related revenues. For instance, all the revenue generated from broadcasting deals is shared equally among all teams, and a significant share of net gate income goes to the visiting team. Even licensing deals, such as income generated from jerseys and posters, is shared. In the NBA, all teams contribute annually a fixed percentage of their total local revenue, roughly 50%, into a revenue-sharing pool. Each team then receives an allocation equal to the league's average team payroll for that season from the revenue pool. In the MLB all teams share

[8]All major sports leagues use a similar type of regular season schedule with a playoff tournament after the regular season ends. The best teams in the regular season reach the playoffs, and the winner of the playoffs is crowned champion of the league

[9]National TV rights are sold collectively by the league, and all Big Four leagues have launched a network of their own, NBA TV in 1999, the NFL Network in 2003, the NHL Network in Canada in 2001, and in the US in 2007, and the MLB Network in 2009. In all leagues but the NFL, individual teams negotiate with local broadcasters to air most of their games (NFL teams do not negotiate local broadcast contracts, but are allowed to negotiate their own television deals for pre-season games).

national broadcasting revenue equally and contribute 34% of their local TV revenue into a shared fund which is divided equally among all teams, and teams can keep all other revenue for themselves (this leads to an estimate of teams sharing approximately 15% of all revenues).[10] The NHL has a revenue sharing program that allocates 6% of total league revenue, primarily away from the top 10 revenue-generating teams, to financially struggling teams. Thus, the effective revenue sharing tax would be above 6%.[11]

Closely related to revenue sharing is the "luxury tax" (sometimes called a competitive-balance tax). This is a payroll tax that typically only kicks in when the total payroll of a team exceeds a predetermined threshold, thus there is a tax levied on money spent above a predetermined limit set by the corresponding sports league.[12] For every dollar a team spends above the tax threshold, those exceeding the limit must also pay some fraction to the league. The money derived from this tax is distributed among the teams with smaller payrolls. The first luxury tax in professional sports was introduced in 1996 by MLB as part of its Collective Bargaining Agreement (CBA). This luxury tax forces MLB teams with high payrolls to pay a dollar-for-dollar penalty. These funds go into a central MLB fund and it is used for marketing programs. In 1999, the NBA also introduced a luxury tax.

While revenue sharing and luxury taxes affect teams incentives to hire players, a more forceful institution that affects hiring is a "salary cap". This is a limit on the amount of money a club can spend on players' salaries that is negotiated in CBAs between players' unions and team owners. The cap is usually defined as a percentage of average annual revenues and limits a club's investment in playing talent. In 1984, the NBA became the

[10]NFL revenue sharing was obtained from http://money.cnn.com. NBA revenue sharing was obtained from http://sportsbusinessdaily.com. MLB revenue sharing was calculated using data from http://awfulannouncing.com and http://forbes.com.

[11]A maximum of 50% of the redistribution commitment is drawn from the top 10 highest-grossing teams based on pre-season and regular season revenue. Each team's contribution is based on how much they earn over and above the 11th-ranked team (implying that the teams in the 8-10 spots contribute less than the top three). NHL revenue sharing was obtained from http://ontheforecheck.com.

[12]See Dietl et al. (2010).

first league to introduce salary cap provisions.[13] Salary caps can be either hard or soft. Under a hard salary cap, the league sets a maximum amount of money allowed for player salaries, and no team can exceed that limit. The NFL and the NHL currently have hard salary caps. A soft salary cap sets a limit to players' salaries, but there are exceptions that allow teams to exceed the cap. In the NBA, for example, teams can exceed the salary cap when keeping players that are already on the team (Dietl et al., 2010). MLB has no salary cap.

Finally, another common institution is the "draft". This is a process used to allocate certain players to teams. In a draft, teams take turns selecting from the pool of new players that want to start playing in the league, with the order being determined (partly) by teams performance last season so that worse performing teams go first. A draft could therefore affect teams incentives to hire good players because good performance one year affects draft order the next year.[14] Together with the "player reservation" system, the draft performed similar functions as revenue sharing as it distributed income from strong to weak teams through players' sales of the latter to the former. Free agency, which was first introduced in 1976 in MLB and shortly afterwards in the other sports, undermined the impact of the draft as an institution that affects the allocation of talent across teams. Thus, we abstract from it in our work.[15]

[13]For details, see Fort and Quirk (1995), Szymanski (2003), and Vrooman (1995)

[14]The effect of a draft on teams incentive to hire skills depends on what is assumed about wage setting. If wage setting is competitive, the order in which teams get to choose the specific players is unimportant because better players will also receive higher wages. If wages are not competitive, however, the draft order may matter, which will affect teams incentives to hire good players because good performance one year affects draft order the next year. Whether the draft increases or decreases incentives to hire good players is still unclear, however, and depend on whether early or late draft picks offer "better deals" relative to the wages they are paid. Conventional wisdom seems to assume that picking early in the draft allows teams to get better players at a lower cost, however, empirical work by Massey and Thaler (2012) have suggested that in the NFL draft, players drafted later actually offer greater value relative to the salaries they are paid.

[15]The reserve clause was devised in the National Baseball League in 1879 to restrict competition on hiring of superstar players. Each team in the league was allowed to "reserve" five of its players, implying that other owners could not attempt to hire them in the end-of-season market. On the revenue sharing

Table 1: Regulations across U.S. sports leagues

League	Revenue shared (%)	Salary cap	Luxury tax
NFL	61	hard	no
NBA	50	soft	yes
NHL	6	hard	no
MLB	15	no	yes

Table 1 summarizes the different regulatory institutions in the Big Four US leagues. A fairly clear ranking of the four leagues arises in terms of the amount of regulatory institutions affecting teams hiring of players. The NFL has the strongest regulatory institutions, followed by the NBA and the NHL. MLB has the least regulatory institutions.[16]

3 The Model

3.1 Teams and competition

The league consists of N ($N \geq 2$) teams that compete in a tournament. Teams play against each other n times, which determines the number of games (or season length), $n(N-1)$. We take N as exogenous, and most of the analysis will consider $N = 2$ and $n = 1$. Since season length may play a role in increasing league-wide profits we explicitly allow for n to be a variable that can be changed by the league authority.[17]

The role of teams in a league is to hire, and coach, players. Following the sports literature, we model this by assuming that teams invest in a one-dimensional input called skill or talent, S, that will influence their probability of winning as well as the entertainment

implications of the reservation and draft system see Fort and Quirk (1995).

[16] As a check on this ranking, we informally polled several known sports economists regarding the regulatory rank of the Big Four leagues. The answers we received consistently confirmed the ranking, with the only slight point of disagreement being the relative standing of the two "in-between" leagues, NBA and NHL.

[17] See Christenfeld (1996) for a critical comparison of season length across several sports.

value of each game. Teams choose S to maximize expected profits, taking league rules and institutions, and other teams' hiring decisions as given.

3.2 Team revenue

Following the sports literature we assume that team i's revenue is proportional to the (exogenously given) size of its local fan base or population, F_i, and to the number of games played, $n(N-1)$, while also depending on the team's probability of winning the tournament, w_i, as well as the quality of the league, captured by the aggregate amount of talent of competing teams, $\bar{S} \equiv \sum_{i=1}^{N} S_i$, through some function $R(\bar{S}, w_i)$. Total team revenue is thus $n(N-1)F_i R(\bar{S}, w_i)$. We make the following standard assumptions on the revenue function, R:

Assumption 1.

$$\frac{\partial R}{\partial \bar{S}} \geq 0, \quad \frac{\partial^2 R}{\partial \bar{S}^2} < 0, \quad \lim_{\bar{S}\to\infty} \frac{\partial R}{\partial \bar{S}} = 0$$
$$\frac{\partial R}{\partial w_i}|_{w_i=\frac{1}{N}} > 0, \quad \frac{\partial^2 R}{\partial w_i^2} < 0, \quad \frac{d^2 R}{dS_i^2} < 0.$$

This reduced-form revenue function can be rationalized as capturing two sources of income to teams, as in Vrooman (1995). First, there is the prize that the winner of the championship earns, and therefore increases with win probability. This could include direct rewards as well as the monetary value of qualifying for the playoffs. Second, there is broadcasting revenue. This increases with the overall quality of the league which is why revenue increases with $\bar{S}$.[18]

The broadcasting revenue, however, also depends on the "suspense" of the outcome. As w_i gets too large, suspense obviously goes down, which is why the function is strictly concave in win probability.[19] We allow this second effect to possibly dominate, and

[18]See Falconieri et al. (2004).

[19]Several empirical studies have documented that the demand for match tickets peaks at a point in which the home team is about twice as likely to win as the away team. See Szymanski (2003) for a survey of these results, as well as inconclusive evidence on seasonal uncertainty. In a recent contribution, Bizzozero et al. (2016) use data from professional tennis and report that both suspense and "surprise" positively affect live TV audience figures.

thus only require that the marginal effect of win probability be positive when team i's probability of winning, w_i, is average, $\frac{1}{N}$. To guarantee a unique solution to teams' optimization problem we require that the revenue function be concave in own skills.

For tractability we assume the following revenue function to derive some results.[20]

Assumption 2.

$$R(\bar{S}, w_i) = \bar{S}^{\sigma} + K \log(w_i), \quad \sigma < 1.$$

Where $K \equiv k \left[\frac{\sigma(\sum_{n=1}^{N} F_i)}{c}\right]^{\frac{\sigma}{1-\sigma}}$, with k measuring the relative importance of win probability in revenue relative to league quality (K reflects a convenient normalization), and c the per game cost of talent.[21]

We make the following assumption on the distribution of fan bases, where we order them from highest, 1, to lowest, N.

Assumption 3.

$$\ln(F_i) = A - \ln(i).$$

Thus we assume that fan bases follow a power law with exponent -1, such that the team with the largest fan base has twice as many fans as the second largest team, three times the number of fans of the third largest team and so on. If we associate a team's fan base to the population of the city where it is located, the postulated distribution is just an instance of Zipf's law (see Zipf (1949)), such that A is the logarithm of the population of the largest metropolitan area in the US.[22]

Under assumption 3, the ratio of fan bases of teams ranked 8^{th} and 23^{rd}, which is a convenient measure of dispersion of fan base for a league with 30 teams, would be $\frac{23}{8} = 2.875$. Using actual population data for the different US sports we find that this

[20]We will explicitly point out which results require assumption 2. Importantly, our main results in proposition 5 do not require it. Note that whether this revenue function satisfies $\frac{d^2R}{dS_i^2} < 0$, as required in assumption 1, depends on the way in which S_i affects w_i. See lemma in the online appendix for an implicit parameter restriction that guarantees concavity in own skills.

[21]The reason for this normalization will be clear after we solve for the efficient allocation.

[22]According to the United States Office of Management and Budget (OMB) this would be New York-Newark-Jersey City with an estimated population in 2015 of 20,182,305.

ratio is 2.76 for NFL, 2.56 for NBA, 3.08 for NHL, and 2.36 for MLB. Thus, the distribution of fan bases accords reasonably well with Zipf's law.

3.3 The supply of talent

The sports literature usually assumes that there is a fixed exogenous supply of skills. We deviate from this assumption for two reasons. First, it seems highly questionable in modern sports, especially in the long run. The share of people participating in competitive sports has increased significantly over time, suggesting that the total supply of skilled players is not fixed over time.[23] Moreover, the observation of a large number of foreign players in many sports leagues suggests that the pool of talent is not fixed at the national league level in the short run.[24] Second, for studying institution choice, it is important that different institutions have an effect on the total amount of talent hired by teams, as this is a significant determinant of aggregate profits. This is of course precluded if the supply of talent is exogenously fixed.[25]

Instead of a fixed supply of skills, we will here assume a perfectly elastic supply such that teams can hire as much talent as they want at a constant per game cost of c. To be clear, this assumption ignores many features of real world labor markets. In practice, the labor market for sport talent has many complications, as players are obviously indivisible and wages typically being set through complicated bargaining with individual players and/or player unions. While the assumption of a constant marginal cost of talent simplifies the analysis, it is not essential to the main mechanisms at work in our model or the empirical results we present later. The only requirement we need is that the complex hiring process is in the aggregate well approximated by a constant cost per unit of talent across teams at a given point in time.

[23]See Rossi and Ruzzier (forthcoming).

[24]The proportion of foreign players is approximately 30% in the MLB and the NBA, 25% in the NHL (considering Canadian players as nationals), and 3.5% in the NFL.

[25]We also note that El-Hodiri and Quirk (1971) and Fort and Quirk (1995) show that, with an exogenous supply of skills, revenue sharing has no impact on the *distribution* of skills, and only reduces players' salaries.

3.4 Win probability

The second deviation of our model from the standard assumptions in the sports literature is on how teams' talents are translated into winning probability. Usually the probability of winning the *championship* is related to the distribution of skills across teams. Since we are interested in examining how differences in the fundamental rules of different sports may affect the choice of institutions, we will instead model the probability of winning *individual games.*

In particular, we assume that the probability that team i defeats team j in a match is given by:

Assumption 4.

$$w_{ij} = \frac{S_i^\alpha}{S_i^\alpha + S_j^\alpha}.$$

This formulation has been widely used in the contest literature.[26] Skaperdas (1996) shows that assumption 4 is the only formulation that depends on the ratio of skills and satisfies a series of basic axioms, including independence of irrelevant alternatives. The parameter α reflects how much skills matter for the win probability. We therefore refer to it as the *productivity of skills.* More formally, since the odds of winning is given by the ratio of the probability of winning to the probability of losing, α corresponds to the elasticity of the odds of winning to the ratio of skills $\frac{S_i}{S_j}$.

The parameter α will depend on the fundamental rules of the sport that teams are playing. As an extreme and trivial example, if the sport being played simply involved flipping a coin to declare a winner, α would equal 0 as the win probability would always be a half. Conversely, if the sport involved comparing players average height, α would equal infinity as the team with more talent (taller players) would always win for sure. In reality the level of α depends on many details of the rules, such as the number of times each team gets to be on the offensive in a given game and the amount of randomness involved in scoring. In section 4.3 we estimate this elasticity for each of the major four

[26]For applications in the sports literature for the probability of winning the championship see Szymanski (2003).

North American sports and show that it differs significantly, going from 0.16 for baseball to 1.54 for basketball.

It is worth noting that the previous literature on comparing different sports usually focuses on ex post outcomes, such as the probability of winning the championship, or the dispersion of wage bills across teams. These measures are clearly influenced by the institutions adopted by the leagues and therefore do not represent a fundamental characteristic of the corresponding sports.[27] In contrast, the elasticity of the odds of winning to the ratio of skills describes a characteristic of sports that is practically invariant to their corresponding institutions. In section 4 we estimate the productivity of skills across sports using team payroll as a measure of skills.[28]

Given the above formulation for the probability of winning a game, we assume that the team that wins the championship is simply the team that wins the most games each season.[29] The probability of winning the tournament, which determines revenue, is then given by a complex relation between the skills of team i, skills of other teams, parameter α, and the number of times teams play against each other, n. For now, we simply represent this relation by[30]

$$w_i = W(S_i, \vec{S}, \alpha, n).$$

3.5 Team's hiring decision without regulatory institutions

In the "laissez-faire" case, when there are no regulatory institutions in place, each team decides how much talent, S_i, to hire by maximizing profits, $\pi_i(S_i)$, taking as given the amount of talent hired by the other teams, S_j:

$$\max_{S_i} \ \pi_i(S_i) = n(N-1)F_i R(\bar{S}, W(S_i, \vec{S}, \alpha, n)) - cn(N-1)S_i.$$

[27]For example, the dispersion of wage bills is affected by the degree of revenue sharing. The probability of winning the league is also a function of season length (see Christenfeld (1996)).

[28]There is strong empirical evidence that wage bills are significant predictors of playing success in different sports. See Simmons and Forrest (2004) and Szymanski and Kuypers (1999).

[29]In practice of course, the leagues we consider use a play-off format. We abstract from this and consider qualifying to the playoffs as the reward to "winning" teams.

[30]We use vector notation such that $\vec{S} \equiv (S_1, S_2, \ldots, S_N)$.

Under assumption 2, the first-order condition for team i is

$$\sigma \left(S_i + \sum_{j \neq i} S_j \right)^{\sigma - 1} + K \frac{1}{w_i} \frac{dw_i}{dS_i} - \frac{c}{F_i} \leq 0. \tag{1}$$

This first-order condition holds as an equality if $S_i > 0$.

3.6 League's objective and efficient allocation

We assume the league wants to maximize aggregate expected profits. Since we want to allow for n to be a choice variable, we introduce costs of setting up and organizing the league, $X(n)$, which we assume are an increasing and convex function of n.[31] Aggregate expected profits is therefore the sum of team revenues minus the setup and organizing costs.

$$\sum_i \pi_i (S_i) - X(n).$$

If a planner were able to perfectly dictate teams' choice of skills $\vec{S}$, it would solve the following problem:

$$\begin{aligned} \max_{n, \vec{S}} \quad & \sum_{i=1}^{N} \pi_i (S_i) - X(n) \\ &= n(N-1) \sum_{i=1}^{N} F_i R(\bar{S}, W(S_i, \vec{S}, \alpha, n)) - cn(N-1)\bar{S} - X(n). \end{aligned}$$

Note that we can reparameterize the planner's problem as choosing n, $\bar{S}$, and $N-1$ win probabilities, w_i. We now characterize the profit maximizing league-wide allocation of talent.

Proposition 1. Under mild concavity assumptions on total revenue (a sufficient condition being $\frac{\partial^2 R}{\partial \bar{S} \partial w_i} = 0$) there is a unique profit maximizing outcome that solves the planner's problem, $(n^P, \vec{S}^P)$. Furthermore $\bar{S}^P \equiv \sum_{i=1}^{N} S_i^P$, and w_i^P do not depend on α or n.

Proof. See appendix 6.1. □

[31] Since teams take n as given, the fixed costs will never matter for individual teams' decisions, which is why $X(n)$ is omitted from the team's problem.

We make the following assumption, such that an unconstrained planner would choose $n^P = 1$.

Assumption 5.
$$\sum_{i=1}^{N} F_i R(\bar{S}^P, w_i^P) - c\bar{S}^P - \frac{dX(n)}{dn}|_{n=1} < 0.$$

This assumption highlights the role of increasing season length to provide teams with adequate hiring incentives under certain conditions.

3.7 Decentralized equilibrium allocation

Henceforth we will restrict the analysis to a two-team league.

Assumption 6. $N = 2$.[32] Assuming n odd, this implies

$$w_1 = W(S_1, S_2, \alpha, n) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} w_{12}^k (1 - w_{12})^{n-k},$$

with w_{12} given by assumption 4, $w_{21} = 1 - w_{12}$, $\frac{\partial w_{12}}{\partial S_1} = -\frac{S_2}{S_1}\frac{\partial w_{12}}{\partial S_2}$, and $\frac{\partial w_{12}}{\partial S_1} = -\frac{\partial w_{21}}{\partial S_1}$.

We now turn to comparing the decentralized outcome to the allocation that maximizes league-wide profits. We start by showing existence and uniqueness of the Nash equilibrium.

Proposition 2. Under assumption 2, there exists a unique Nash equilibrium.

Proof. See online appendix. □

A useful way of understanding whether and how the decentralized equilibrium differs from the profit maximizing league-wide allocation is to examine the first-order conditions for the planner and team 1 with respect to S_1. The first order condition for the planner is given by:

$$F_1 \frac{\partial R(\bar{S}, w_1)}{\partial \bar{S}} + F_2 \frac{\partial R(\bar{S}, w_2)}{\partial \bar{S}} + F_1 \frac{\partial R(\bar{S}, w_1)}{\partial w_1} \frac{dw_1}{S_1} - F_2 \frac{\partial R(\bar{S}, w_2)}{\partial w_2} \frac{dw_1}{S_1} = c,$$

[32]Note that under assumption 3, this implies $F_1 = 2F_2$.

The first order condition for team 1 is given by:

$$F_1 \frac{\partial R(\bar{S}, w_1)}{\partial \bar{S}} + F_1 \frac{\partial R(\bar{S}, w_1)}{\partial w_1} \frac{dw_1}{S_1} = c.$$

Comparing the first-order conditions for the team's problem to the first-order conditions for the planner's problem, we see that they are the same, except that the marginal profit from hiring an additional unit of skill includes two additional terms in the planner's case: $F_2 \frac{\partial R(\bar{S}, w_2)}{\partial \bar{S}}$ and $-F_2 \frac{\partial R(\bar{S}, w_2)}{\partial w_2} \frac{dw_1}{S_1}$. These two terms reflect two externalities that teams fail to incorporate when choosing skill investments. The first is positive and reflects the fact that when team 1 increases its skill level, this increases team 2's revenue through its effect on total league talent. This creates a *common pool* problem. The failure of team 1 to internalize this externality implies that it will tend to hire *less* skill than what is optimal for the league as a whole. The second term is negative and reflects the fact that when team 1 employs more skill and raises its own win probability this lowers the win probability of team 2 because winning is a *zero sum* game. The failure of team 1 to internalize this externality implies that it will tend to hire *more* skill than what is optimal for the league.

Importantly, the strength of the zero sum externality, and thus a team's incentive to hire skills, depends crucially on the productivity of skills, α. In particular we have the following proposition.

Proposition 3. There exists $\underline{\alpha}(F_1, F_2) > 0$, with $\lim_{F_2 \to \infty} \underline{\alpha}(F_1, F_2) = 0$, such that for $\alpha > \underline{\alpha}(F_1, F_2)$ the equilibrium levels S_1^* and S_2^* are increasing in α. When $w_{12}^* < \overline{w_{12}}(n) \equiv \frac{1+\sqrt{\frac{1}{n+2}}}{2}$, equilibrium levels S_1^* and S_2^* are increasing in n.

Proof. See appendix 6.2. □

Proposition 3 highlights the first insight of the model: under reasonable assumptions (i.e. when $\alpha > \underline{\alpha}(F_1, F_2)$), teams' hiring incentives are increasing in the productivity of skills, α. The intuition behind the result is simple. When α is high, the effect of an additional unit of skills on the probability of winning, and thus on revenue, is stronger. The reason for the restriction $\alpha > \underline{\alpha}(F_1, F_2)$ has to do with the tail behavior of the win probability function when one of the skill levels is close to zero (see appendix 6.2).

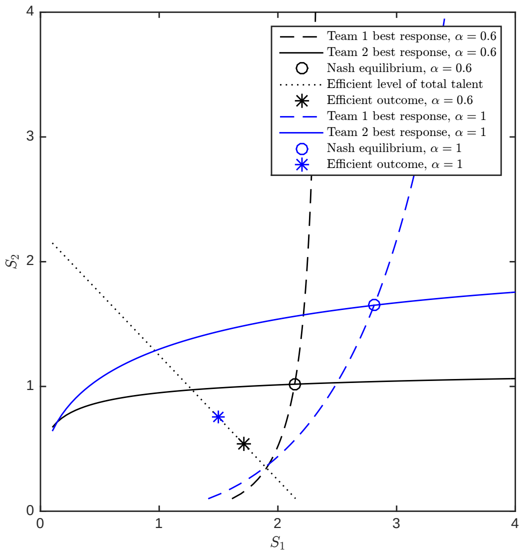
The second part of the proposition characterizes the effect of increasing season length, n. Since $S_1^* + S_2^*$ is increasing in α we denote by $\bar{\alpha}(n)$ the value of parameter α that equates teams' aggregate skills to the profit-maximizing level $\bar{S}^P$, for a given n.[33] Thus, for $\underline{\alpha}(F_1, F_2) < \alpha < \bar{\alpha}(n)$ the decentralized equilibrium has less talent than the profit-maximizing allocation, and conversely for $\alpha > \bar{\alpha}(n)$. When $w_{12}^* < \overline{w_{12}}(n)$ (see online appendix), $\frac{d\bar{\alpha}(n)}{dn} < 0$, meaning that an increase in season length increases $S_1^* + S_2^*$. The intuition behind this is that as teams play more and more games against each other, by the law of large numbers it becomes less and less likely that an inferior team gets lucky and wins the majority of the games. Thus, an increase in season length has the effect of increasing the "effective" productivity of skills, thereby increasing hiring incentives. But the effect of increasing season length is limited: as the probability that the strongest team wins gets closer to 1, the marginal incentive to hire decreases. This is reflected in $\overline{w_{12}}(n)$ decreasing in n.

Figure 1 provides numerical results that illustrate the main insights of Proposition 3. Using the functional form from assumption 2 and a given set of other model parameters, it shows the teams' best response functions under the associated Nash equilibrium as well as the efficient allocation when $\alpha = 0.6$ and $\alpha = 1$. From the figure, we see that at these parameters, $\alpha = 0.6$ is high enough that the "zero sum" externality dominates and the Nash equilibrium involves both teams choosing higher levels of skills than the efficient outcome. As discussed above, a higher α strengthens the "zero sum" externality, which causes the best response functions to shift outward and upward, thereby increasing the equilibrium level of skills for the two teams. In contrast, the efficient level of total talent is independent of α so the efficient allocation only moves along the negatively sloped 45-degree line.

[33] Note that $\bar{\alpha}(n) = \infty$ indicates that teams would never choose more talent than the level that maximizes aggregate profits.

Figure 1: Decentralized Nash equilibria and efficient outcomes

Model outcomes for $\alpha = 0.6$ and $\alpha = 1$:



The graph shows numerical results from the model under assumption 2. Solid and long-dashed lines shows teams' best response functions and circles show Nash equilibria. Stars show efficient allocations, while the short-dashed lines shows allocations involving the efficient level of total talent. Note that the efficient level of total talent is independent of α. The additional parameter values used are $F_1 = 2$, $F_2 = 1$, $\sigma = 0.5$, $n = 1$, $K = 2$ and $c = 1$.

3.8 The role of regulation and redistributive taxation

In the previous section, we saw that the decentralized Nash equilibrium in the league generally does not maximize total league profits as teams hire too much (or too little) skill. This suggests that the league may centrally want to introduce regulatory institutions that affect hiring incentives.

We will focus on revenue sharing since it is the canonical example of a redistributive, regulatory institution in team sports. In addition, we assume the league can introduce a salary tax or subsidy that only affects one team. The motivation for this is that league revenue depends on choices by both teams, thus it is generically not possible to implement the profit maximizing allocation using only a single policy instrument. Introducing an asymmetric hiring tax or subsidy solves this issue allowing us to characterize first-best institutions.[34]

Following the literature we introduce revenue sharing by assuming that each team keeps $1 - \tau$ of its own revenues and receives $\frac{\tau}{N}$ of aggregate team's revenues (including own revenue). Since sports leagues always have a relatively modest amount of teams we also follow the sports literature in assuming that teams are not myopic but take into account that their decisions affect aggregate shared league revenues.[35] For the weak team we introduce a hiring subsidy at rate s financed by lump sum contributions or "taxes"

[34]For the same reason, in a two-team league other institutions that directly or indirectly affect teams' hiring of talent are likely to implement the profit maximizing allocation.

[35]It is worth noting that under this non-myopic assumption, an easy way for the planner to always implement the efficient outcome is to implement full profit sharing. With non-myopic teams and full profit sharing each team will choose their own skill so as to maximize league profits. We view full profit sharing as infeasible in practice, however, because teams in the real world can spend money on things other than talent. Under full profit sharing, teams would therefore have the incentive to incur unnecessary costs to lower their own profits. For example, the owner may hire a relative into an overpaid consultant position.

from both teams.[36] In this case, team i's problem becomes:

$$\max_{S_i} \pi_i(S_i) = \left(1 - \frac{\tau}{2}\right) F_i R(S_i + S_j, w_i) + \frac{\tau}{2} F_j R(S_i + S_j, w_j) - c(1 - s\mathbb{1}_{i=2})S_i - T_i,$$

where the indicator function tells us that for team 2 the per unit cost of skills is now $c(1-s)$, and T_i are the lump sum taxes used to pay for the hiring subsidy, $T_1 + T_2 = csS_2^*$. We are interested in examining how regulations affect total league revenue. We begin by establishing that the model has a unique Nash equilibrium when regulatory institutions are introduced.

Proposition 4. Under assumption 2, the model has a unique Nash equilibrium.

Proof. See online appendix. □

Next we analyze the effect of regulations on team behavior and their role in increasing total league profits. The first-order condition for team i is:

$$\left(1 - \frac{\tau}{2}\right) F_i \left(\frac{\partial R}{\partial S_i} + \frac{\partial R}{\partial w_i}\frac{dw_i}{dS_i}\right) + \frac{\tau}{2} F_j \left(\frac{\partial R}{\partial S_i} + \frac{\partial R}{\partial w_j}\frac{dw_j}{dS_i}\right) - c(1 - s\mathbb{1}_{i=2}) \leq 0. \quad (2)$$

Comparing this new first-order condition with the one without regulations, we see that the terms corresponding to the effect of additional skill on the other team now appear, although only with a weight of $\frac{\tau}{2}$. In addition, the terms corresponding to the effect of additional skill on team i's own revenue now only enter with a weight of $1 - \frac{\tau}{2}$. Intuitively this shows that revenue sharing has two effects: a) it gets teams to partially internalize the externalities from before, and b) it tends to lower the incentive to invest in skill because teams now only get to keep $1 - \frac{\tau}{2}$ of own revenue. The hiring subsidy increases team 2's incentive to hire skills (or decreases it if the subsidy is negative), while having no effect on team 1.

When the decentralized equilibrium involves teams choosing too high skill levels ($\alpha > \bar{\alpha}(1)$), this suggests that (higher levels of) revenue sharing can improve league profits by

[36] We allow this subsidy to be negative, i.e. that the weak team be taxed. In 6.3 we characterize parametric conditions for the subsidy to be positive. We choose a hiring subsidy for the weak team instead of a hiring tax on the strong team because this imposes less restrictions on the productivity of skills for proposition 5 to hold. See 6.3.

causing teams to lower the amount of skill they hire in equilibrium. In particular we have the following proposition.

Proposition 5. When $\alpha > \bar{\alpha}(1)$ there exists a policy $(\tau^*(\alpha), s^*(\alpha))$ that leads to the profit maximizing allocation, and $\frac{d\tau^*(\alpha)}{d\alpha} \geq 0$. An increase in α reduces the dispersion of talent across teams. When $\alpha < \bar{\alpha}(1)$, and $w_{12}^P < \overline{w_{12}}(1) \equiv \frac{1+\sqrt{\frac{1}{3}}}{2}$ (the latter satisfied under assumption 2), the league has an incentive to increase season length.

Proof. See appendix 6.3. □

Proposition 5 shows that regulatory institutions can achieve the profit-maximizing outcome if sports have a sufficiently high α. According to proposition 3, when $\alpha > \bar{\alpha}(1)$ teams choose inefficiently high levels of skill. Revenue sharing and a hiring subsidy to the weak team lead to the efficient allocation. Moreover, leagues for sports that involve higher levels of α will optimally want to introduce higher levels of revenue sharing to counter the externalities discussed above. This is the second main insight of our model. In other words, in sports where teams' win probabilities are more sensitive to relative skill, teams have inefficiently strong incentives to hire talent. Leagues authorities in these sports therefore choose to introduce higher levels of revenue sharing to dampen teams' individual incentives.

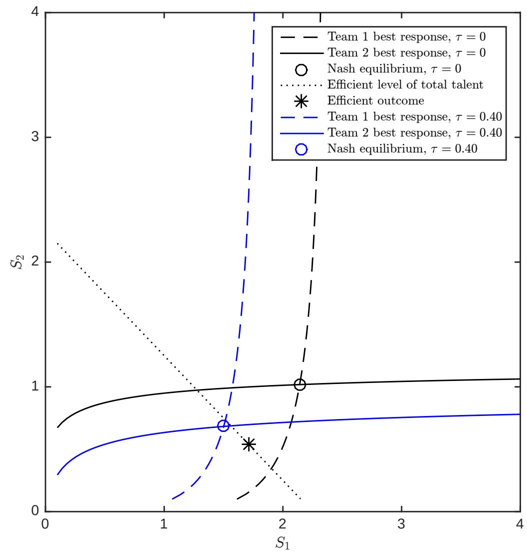
Proposition 1 showed that, conditional on fan bases, all sports have the same profit-maximizing win probability, w_1^P. Since this allocation is attainable to all sports with $\alpha > \bar{\alpha}(1)$, and given that $\frac{w_{12}}{w_{21}} = \frac{S_1^\alpha}{S_2^\alpha}$, sports with a higher α tend to produce more dispersed win probabilities for a given distribution of talent. Thus, as the productivity of skills is increased, to attain w_1^P talent has to be more evenly distributed across teams.

The last part of the proposition highlights that season length n, may also play a role. If $\alpha < \bar{\alpha}(1)$, teams are initially hiring less than the optimal amount of skills, which can only be exacerbated by introducing revenue sharing. In this case, however, the league can increase teams' incentives to hire talent by increasing season length and then introduce regulatory institutions to increase league-wide profits. Because this increases organization costs the equilibrium might feature $S_1^* + S_2^* < \bar{S}^P$.[37]

[37]Due to indivisibilities in season length, if n^P is such that $\alpha > \bar{\alpha}(n^P)$, the analysis of proposition 5

Figure 2: Effects of introducing revenue sharing

Improving efficiency with revenue sharing, $\alpha = 0.6$*:*

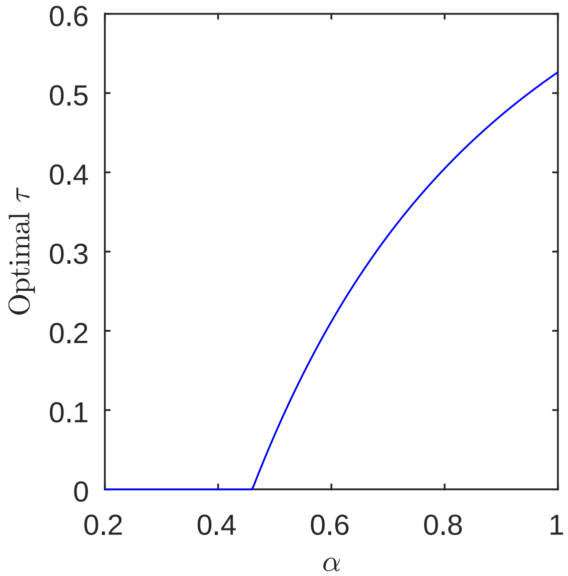


The graph shows numerical results from the model. Solid and long-dashed lines shows teams' best response functions and a circles show Nash equilibria. A star shows the efficient allocation. Black lines correspond to outcomes without revenue sharing, while blue lines correspond to outcomes with 40% revenue sharing. The additional parameter values used are $F_1 = 2$, $F_2 = 1$, $\sigma = 0.5$, $n = 1$, $K = 2$, $c = 1$ and $s = 0$.

Figure 2 illustrates these points. For a given set of other model parameters and $\alpha = 0.6$, the black lines and circle show the best response functions and the Nash equilibrium without revenue sharing. The black asterisk shows the efficient outcome. The Nash equilibrium has the two teams hiring more talent than what is efficient. There is thus scope for improving efficiency by introducing revenue sharing. The blue lines and markers therefore show the effect of introducing a revenue sharing of 40% ($\tau = 0.40$). As discussed above, revenue sharing reduces team's individual incentives to hire skills and, as a result, shifts the best response functions and the equilibrium level of skills for both teams inwards so that they are much closer to the efficient levels.

goes through, the league employs revenue sharing and $S_1^* + S_2^* = \bar{S}^P$. If $\alpha < \bar{\alpha}(n^P)$, $S_1^* + S_2^* < \bar{S}^P$ and proposition 5 is silent on what other institutions, besides increasing season length and not sharing revenue, are chosen.

Figure 3: Optimal revenue sharing



The graph shows numerical results from the model. The line shows the optimal level of revenue sharing, τ, as a function of α, when the league simultaneously introduces an optimally set wage subside s. The additional parameter values used are $F_1 = 2$, $F_2 = 1$, $\sigma = 0.5$, $n = 1$ and $K = 2$, $c = 1$.

Note that in figure 2 introduction of revenue sharing alone is not enough for the league to implement the efficient allocation. To do this, the league must also introduce an additional instrument that affects hiring decisions. Figure 3 shows how the optimal level of revenue sharing, τ, varies with the level of α assuming that the league also introduces an optimally set hiring subsidy for the weak team. As the figure illustrates revenue sharing is increasing in the productivity of skills when $\alpha > \bar{\alpha}(1)$, and there is no revenue sharing when $\alpha \leq \bar{\alpha}(1)$.

3.9 Summary

The preceding analysis was built on a standard league model from the sports literature. It provides the prediction that, due to externalities, teams will generally not reach the profit-maximizing allocation in the absence of regulatory institutions. Inefficiencies imply that league authorities have the incentive to introduce institutions that affect teams' hiring decision. We showed that the strength and nature of hiring externalities depend crucially on the fundamentals of the sport being played via the productivity of skills, α, such that teams in sports with higher α's will tend to hire more talent. Regulatory institutions such as revenue sharing can implement the allocation that maximizes league profits. Therefore, sports with higher levels of α will require higher levels of revenue sharing.

4 Empirical results

In this section we use data on match outcomes and players' payroll to estimate how the productivity of skills differs across the sports played in the Big Four US leagues. We then relate this to the use of regulatory institutions in the different leagues.

4.1 Data

The first piece of data we need is information about team skills. As discussed in detail in the next section, this data will be based on team's total payroll (skill expenditure). Since our objective is to make comparisons across the different sports' leagues, it is important that the payroll data we use is comparable across the different leagues. For this reason we use salary data from the yearly Global Sports' Salaries Survey (GSSS). The GSSS reports team salaries for a range of different sports leagues, including the NFL, NBA, NHL and MLB. Importantly, the GSSS data is constructed explicitly with the aim of making comparisons across different sports' leagues.

We base our measure of total team skill expenditure on the GSSS definition of player salaries, which includes both players' base salaries and any performance bonuses that have been paid out.[38] From the 2012-2015 GSSS data, we construct total yearly team payrolls

[38]We include bonus payments in our measure of skill expenditures for two reasons: First, the key choice variable in our model is how much teams choose to spend in total on talent, regardless of whether this is paid out as base salary or as performance related bonuses. Second, as a purely practical consideration, we are unaware of any data source on team payrolls which makes it possible to separate out bonus and performance pay in a consistent way across multiple sports. Note that GSSS reports salaries before

for each team for each season.[39]

The second piece of data we use is data on game outcomes in each of the sports leagues. For each league we collected data on all regular season games during the four seasons that correspond to our salary data.[40]

Based on these sources, we construct a data set where each observation corresponds to a particular game. For game g taking place in league l in season t between the home team i and the away team j, we define the $SkillRatio_{tlgij}$ as the ratio of the skill expenditure (payroll) of the home team to the away team. We define the dummy variables NFL_l, NBA_l, NHL_l and MLB_l as indicators for the different leagues. We define the variable $RegulatoryRank_l$ as the ranking of league l in terms of the extent of regulatory institutions such as revenue sharing (where NFL has rank 1, NBA rank 2, NHL rank 3, and MLB rank 4, cf. Table 1). We define the variable $HometeamWin_{tlgij}$ as an indicator for whether the home team i won against the away team j in game g in league l in season t.[41]

Table 2 presents summary statistics of our data.

4.2 Empirical specification and estimation

We now discuss how to take the theoretical model from Section 3 to the data and estimate the relationship between skill input and win probabilities for each sport. To accommodate

the end of the playing season so some performance bonuses, those of the most succesful teams, are not included.

[39]The GSSS reports data on average player salary. We convert this to total team payrolls by multiplying by the typical number of players on a team in each of the four leagues (53 in the NFL, 29 in MLB, 25 in the NHL and 15 in the NBA). To the extent that not all teams within a league carry the same number players on the roster for the full year, this will introduce some measurement error.

[40]For the NFL, NBA and NHL, where seasons run from the Fall of year t to the Spring of year $t+1$, these are: 2011-2012, 2012-2013, 2013-2014, 2014-2015. For the MLB where seasons run from the Spring of year t to the Fall of year t, these seasons are: 2012, 2013, 2014, 2015.

[41]A minor complication arises in defining this variable and taking our theoretical model to the data, due to the possibility of tie games. While our model does not allow for ties, ties are possible in some of the sports we analyze, although they are extremely rare. We present results here where tie games are normalized as wins for the home team. Due to the very few ties in our data, however, dropping ties or instead treating them as losses for the home team leads to virtually identical estimates.

Table 2: Summary statistics

NFL

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Hometeam skill expenditure	1,024	110,710	13,389	59,519	155,820
Skill ratio	1,024	1.016	0.189	0.488	2.051
Log skill ratio	1,024	0.000	0.179	-0.718	0.718
Hometeam wins	1,024	0.578	0.494	0	1

NBA

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Hometeam skill expenditure	4,679	67,473	11,600	33,150	102,150
Skill ratio	4,679	1.034	0.276	0.354	2.828
Log skill ratio	4,679	0.000	0.258	-1.040	1.040
Hometeam wins	4,679	0.588	0.492	0	1

NHL

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Hometeam skill expenditure	4,116	61,353	8,384	38,000	83,500
Skill ratio	4,116	1.018	0.196	0.502	1.993
Log skill ratio	4,116	0.000	0.190	-0.690	0.690
Hometeam wins	4,116	0.548	0.498	0	1

MLB

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Hometeam skill expenditure	9,720	109,893	41,870	23,780	232,870
Skill ratio	9,720	1.167	0.726	0.115	8.720
Log skill ratio	9,720	0.000	0.554	-2.166	2.166
Hometeam wins	9,720	0.536	0.499	0	1

The table shows summary statistics for key variables for each of the sports leagues: NFL, NBA, MLB, and NHL. The unit of observation is a game. Hometeam skill expenditure is measured in 1,000s of dollars. Differences in the number of observations across the different leagues reflect differences in typical number of games per season as well as idiosyncratic differences in the number of games played across seasons for example due to player strikes.

the fact that we now consider four different leagues and use data from multiple seasons and games, we adapt the model setup and notation from Section 3 as follows: We let c_{tl} denote the cost of talent in league l during season t. We let S_{tli} denote the skill employed by team i from league l during season t. We let w_{tlgij} denotes the probability that team i beats team j in game g during season t in league l. Finally, we let α_l denote the returns to skill in league l.

With this notation, the win probability in a given game depends on skill inputs as follows:

$$w_{tlgij} = \frac{S_{tli}^{\alpha_l}}{S_{tli}^{\alpha_l} + S_{tlj}^{\alpha_l}}. \tag{3}$$

Conveniently for the empirical implementation, simple algebra shows that this formulation is equivalent to a standard logit model:

$$w_{tlgij} = \text{logistic}\left(\alpha_l \log\left(\frac{S_{tli}}{S_{tlj}}\right)\right). \tag{4}$$

We will use this logit formulation to estimate the different α_l's from the data described in Section 3. We can relate w_{tlgij} to the variables in our data as follow:

$$P\left(HometeamWin_{tlgij}\right) = w_{tlgij}. \tag{5}$$

For the ratio $\frac{S_{tli}}{S_{tlj}}$, we note that under the assumption that costs, c_{lt}, are constant across teams in a given league and season, the ratio of employed skills is simply equal to the ratio of skill expenditures from our data.[42]:

$$\frac{S_{tli}}{S_{tlj}} = \frac{c_{tl}S_{tli}}{c_{tl}S_{tlj}} = SkillRatio_{tlij}. \tag{6}$$

Combining (4), (5) and (6), we see that we can estimate α_l from our data using the following logit model:

$$P\left(HometeamWin_{tlgij}\right) = \text{logistic}\big(\alpha_l \log\left(SkillRatio_{tlij}\right)\big) \tag{7}$$

[42]Note again that we do not require that players receive their full marginal revenue products. We only require the weaker assumption that talent costs are well approximated in practice by a constant per unit cost across teams. Even if teams are able to pay players below the marginal revenue product, this assumption is satisfied as long as the degree of player exploitation is the same across teams.

Rather than estimate this equation individually for each of the four leagues in our data, however, it will be convenient to use dummy variables for the different leagues and formulate one model:

$$\begin{aligned} P\left(HometeamWin_{tlgij}\right) = \text{logistic}\Big(&\alpha_{NFL}\log\left(SkillRatio_{tlij}\right)\times NFL_l \\ &+\alpha_{NBA}\log\left(SkillRatio_{tlij}\right)\times NBA_l \\ &+\alpha_{NHL}\log\left(SkillRatio_{tlij}\right)\times NHL_l \\ &+\alpha_{MLB}\log\left(SkillRatio_{tlij}\right)\times MLB_l\Big) \end{aligned} \tag{8}$$

Estimation of (8) will be our main approach to estimating α_l for the individual leagues.

The conclusion from the theoretical model from Section 3 is that sports with inherently higher levels of α_l tend to adopt more regulatory institutions. A crude way of capturing this in the empirical model is to substitute in $\alpha_l = \bar{\alpha} + \rho \cdot RegulatoryRank_l$ to get:

$$\begin{aligned} P\left(HometeamWin_{tlgij}\right) =& \text{logistic}\Big(\bar{\alpha}\log\left(SkillRatio_{tlij}\right) \\ &+\rho\log\left(SkillRatio_{tlij}\right)\times RegulatoryRank_l\Big) \end{aligned} \tag{9}$$

In this model, ρ measures the relationship between α_l and the degree of regulation in the league. If leagues with higher α_l optimally adopt more regulatory institutions, we should have $\rho < 0$.

We note that the estimating equations above treat the home and away team symmetrically. In practice, however, there is evidence that in many sports home teams are more likely to win. While our theoretical model does not include this, for robustness we will also estimate versions of the logit models above that allow for a league-specific homefield advantage, ψ_l. This simply corresponds to include the un-interacted indicator variables NFL_l, NBA_l, NHL_l and MLB_l as regressors:

$$P\left(HometeamWin_{tlgij}\right) = \text{logistic}\Big(\alpha_{NFL}\log\left(SkillRatio_{tlij}\right) \times NFL_l \qquad (10)$$

$$+\alpha_{NBA}\log\left(SkillRatio_{tlij}\right) \times NBA_l$$

$$+\alpha_{NHL}\log\left(SkillRatio_{tlij}\right) \times NHL_l$$

$$+\alpha_{MLB}\log\left(SkillRatio_{tlij}\right) \times MLB_l$$

$$+\psi_{NFL}NFL_l + \psi_{NBA}NBA_l + \psi_{NHL}NHL_l + \psi_{MLB}MLB_l\Big)$$

$$P\left(HometeamWin_{tlgij}\right) = \text{logistic}\Big(\bar{\alpha}\log\left(SkillRatio_{tlij}\right) \qquad (11)$$

$$+\rho\log\left(SkillRatio_{tlij}\right) \times RegulatoryRank_l$$

$$+\psi_{NFL}NFL_l + \psi_{NBA}NBA_l + \psi_{NHL}NHL_l + \psi_{MLB}MLB_l\Big)$$

4.3 Marginal productivity of skills and regulatory institutions

We now turn to estimating α_l for the different sports using the logit models from above. Table 3 presents estimates of the logit models in equations 8-11. Because each observation in our data pertains to a pair (dyad) of teams playing against each other, and because team outcomes may be correlated across seasons, we use dyadic cluster robust standard errors throughout and cluster at the team-level.[43]

Column (1) presents estimates of a logit model where the explanatory variables are interactions between the log salary ratio and dummy variables for the four different sports. This corresponds to equation (8) from above and accordingly the estimated coefficients on each of the interaction terms corresponds to estimates of α_l for each of the sports. We see substantial differences across the four sports as $\hat{\alpha}_l$ ranges from 0.16 to 1.54. Moreover we can strongly reject the hypothesis of a constant α ($p < 0.001$).

Looking at the magnitudes of the estimated α_l's in Column (1), we also note that they accord very well with the prediction of our theoretical model regarding league regulation

[43] See Cameron and Miller (2014) and Aronow et al. (2015). There are 122 teams (clusters) in our data in total.

Table 3: Differences in α_l across sports

VARIABLES	(1) Logit: Hometeam win	(2) Logit: Hometeam win	(3) Logit: Hometeam win	(4) Logit: Hometeam win
Log skill ratio × NFL (α_{NFL})	0.656		0.673	
	(0.560)		(0.576)	
Log skill ratio × NBA (α_{NBA})	1.538***		1.590***	
	(0.361)		(0.374)	
Log skill ratio × NHL (α_{NHL})	0.739***		0.745***	
	(0.201)		(0.200)	
Log skill ratio × MLB (α_{MLB})	0.161***		0.162***	
	(0.0590)		(0.0603)	
Log skill ratio		2.486***		2.564***
		(0.562)		(0.580)
Log skill ratio × Regul. rank (ρ)		-0.579***		-0.599***
		(0.143)		(0.148)
NFL (ψ_{NFL})			0.316***	0.324***
			(0.0516)	(0.0537)
NBA (ψ_{NBA})			0.371***	0.368***
			(0.0286)	(0.0285)
NHL (ψ_{NHL})			0.194***	0.194***
			(0.0297)	(0.0300)
MLB (ψ_{MLB})			0.143***	0.143***
			(0.0230)	(0.0230)
Observations	19,539	19,539	19,539	19,539
p-value, α equal across sports	< 0.001		< 0.001	

The table shows estimates from Logit models with home team victory as the outcome. The explanatory variables used are the log skill expenditure ratio of the home team to the away team, indicators for the different leagues, the regulatory rank of each league, as well as interaction terms between these regressors as shown. Dyadic cluster robust standard errors clustered at the team level are in parenthesis. $* : p < 0.10$, $** : p < 0.05$, $*** : p < 0.01$

Table 4: Pairwise tests of differences across sports

Tests of $H_A : \alpha_l \neq \alpha_k$ vs. $H_0 : \alpha_l = \alpha_k$, p-values:				
k:	NFL	NBA	NHL	MLB
l:				
NFL		0.187	0.889	0.381
NBA			0.055*	< 0.001***
NHL				0.007***
MLB				

The matrix shows p-values from pairwise t-tests of differences in α_l across sports based on the logit model in column (1) of Table 3. The test are based on dyadic cluster robust standard errors clustered at the team level. $* : p < 0.10$, $** : p < 0.05$, $*** : p < 0.01$

and the relative regulatory ranking of the four leagues described in Section 3. For three of the four leagues, NBA, NHL and MLB, the ranking in terms of $\hat{\alpha}$ exactly mirrors the ranking in terms of regulation. Among these leagues, the NBA is the most heavily regulated and also has the highest estimated productivity of skills, 1.54, while MLB is the least regulated and has the lowest estimated productivity of skills, 0.16. Moreover, as shown in Table 4, the estimated differences in productivity across these three leagues are statistically significant at least at the 10 percent level.

Turning to the fourth league, the NFL, the estimated $\hat{\alpha}$ does not fit with the predictions of the model given the NFL's regulatory ranking. The NFL is the most heavily regulated league, however, its estimated productivity of skills is relatively low, only 0.66. At the same time, however, because the NFL has so few games per season and also has the smallest variance in skill expenditures across teams, the standard error on the estimated productivity of skills in the NFL is very large.[44] As a result, we cannot reject that the

[44] Each team in the NFL only plays 16 games each season, whereas the teams in the four other leagues play as least four times as many. Since observations in our data correspond to games, this implies that we have a much smaller sample of data for the NFL. Moreover, the variance in payroll across teams is also smallest in the NFL the sample variance in our main regressor $SkillRatio_{tlgij}$ is also the smallest in

productivity of skills in the NFL is larger than in each of the other sports (see Table 4).[45]

As a succinct way of capturing the relationship between productivity of skills, $\hat{\alpha}_l$ and the amount of regulation, column (2) of Table 4 presents estimates a model that includes the log salary ratio and an interaction term between the log salary ratio and the leagues ranking in terms of regulation, corresponding to equation (9) from above. We find a highly significant negative coefficient on the interaction term ($\rho < 0$); leagues with higher levels of alpha have systematically more regulation (a lower rank).

Finally, columns (3) and (4) of Table 3, check how estimates are affected by allowing for a league-specific home field advantage, as in equations (10) and (11). We see that our estimates are virtually unaffected by these alternative specifications.

Overall, we conclude that the predictions from our theoretical model fit very well with the observed relationship between productivity of skills and amount of regulatory institutions across the four leagues.[46]

4.4 Marginal productivity of skills and season length

The main focus of this paper is the relationship between marginal productivity of skills and regulatory institutions. Our model however also yields predictions about season length. In particular, Proposition 5 highlights that leagues with very low productivity of skills, such that $\alpha \leq \bar{\alpha}(1)$, may need to choose a longer season length in order to implement the

the NFL, which further lowers the precision of our estimates.

[45]Note also that this problem is not one that can readily be solved by additional data collection. Given our current estimates and assuming that standard errors decrease with the square root of the sample size (which due to clustering at the team level is likely an optimistic assumption), we would need to more than double the number of seasons in our data set to be able to reject that α_{NFL} is higher than α_l for the other four leagues at the 5 percent level of significance. Even ignoring the issue of comparability across leagues, we are unaware of any data source that contains consistent payroll data for eight NFL seasons.

[46]Szymanski (2003) presents estimates of the pay-performance sensitivity of these, and other, sports using season winning percentage and relative wage bills. He finds that the pay performance sensitivity is highest in NFL, followed by NBA, NHL and lowest in MLB. While Szymanski's results do not capture a fundamental characteristic of the corresponding sports, it is reassuring that we find the same relative standing for the three leagues for which our estimates are statistically significant.

efficient outcome. Comparing observed season lengths in the four sports leagues with our estimated productivity of skills in the four sports, we find some support for this prediction. In particular, MLB, which has by far the lowest estimated productivity of skills also has by far the longest season length. Teams in the MLB play 162 regular season games as opposed to 82 in the NHL and NBA, and only 16 in the NFL.[47]

5 Conclusions

In this paper, we examined institutional choice across the Big Four US sports leagues. Despite having very similar business models and facing the same economic and legal environment, these leagues exhibit large differences in their use of regulatory institutions such as revenue sharing, salary caps or luxury taxes. Since the four leagues differ in that they play sports with very different rules, it seems natural to associate institutional choice as the optimal response to sports' fundamentals.

Building on a standard model of sports leagues, we showed theoretically that heterogeneity in the characteristics of the underlying sports regarding how skills translate into win probabilities may make it optimal for some leagues to adopt systematically more regulatory institutions. Because they play games against each other, teams' hiring decisions are subject to externalities that may lead to inefficiently high levels of talent. The strength of the externalities depend on the marginal productivity of skills. We find that for sports with a higher marginal productivity of skills, stronger hiring externalities make it optimal for leagues to introduce regulatory institutions that constrain teams' hiring incentives, such as salary caps, revenue sharing or payroll taxes.

Using data on game outcomes and team payrolls we then estimated the marginal productivity of skills for the four US sports leagues and found them to be significantly

[47]At the same time of course, the number of games played in the different leagues likely also reflects other factors such as differences in injury risks and how physically demanding the different sports are. Injury risk likely contributes to the very short season length in the NFL. For example, Hootman et al. (2007) estimates injury rates in collegiate sports, finding them to be by far largest in football, followed by hockey, basketball and baseball.

different, ranging from 0.16 for MLB, to 1.54 for the NBA. Comparing the estimated productivity of skills with the institutions actually observed in the different leagues, we find that the theoretical predictions from the model fit perfectly for three of the four leagues. Overall, the observed differences in adopted institutions is well explained as optimal responses to these differences in the marginal productivity of skills.

Our work highlights some pathways for future research in the field of sports economics. We have here focused on one fundamental parameter of a sport, the productivity of skills, and shown how it relates to the optimal choice of regulatory institutions that constrain teams' hiring decisions. It is natural to try and extend this approach to focus on other fundamental sports' parameters (e.g. sports differ in the way in which suspense and surprise develop over time), as well as possibly other, more complex institutional differences across leagues and over time.

More broadly, we view the results of our case study of US sports leagues as particularly relevant for the economics literature on institutions. A central tenet of this literature, and the related policy debate, is that economic outcomes can be improved by transplanting successful institutions from one context to another. In the specific case of the sports leagues we consider, for example, commentators periodically suggest that one or more of leagues adopt regulatory institutions from the other leagues. Our work suggests that such policy prescriptions may be misguided. When observed differences in institutions reflect optimal responses to differences in underlying fundamentals, then transplanting institutions across settings may in fact be harmful.

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6 Appendix

6.1 Proof of Proposition 1

In solving the planner's problem, it will be convenient to reparameterize it so that the planner directly chooses n, total talent, $\bar{S}$, and win probabilities, w_i $(i = 1, .., N-1)$, instead of choosing n, and team skills S_i. The reparameterized problem is equivalent to the original one because, given n, for any $\vec{S}$ there is a one-to-one correspondence to a set of total skills $\bar{S}$ and win probabilities, w_i, and vice versa.

The reparameterized maximization problem is:

$$\max_{n,\bar{S},\vec{w}} = Q(\bar{S}, \vec{w}; \vec{F}) = n(N-1)\sum_{i=1}^{N} F_i R(\bar{S}, w_i) - cn(N-1)\bar{S} - X(n).$$

First-order conditions imply (using $w_N = 1 - \sum_{i=1}^{N-1} w_i$)

$$\frac{\partial Q}{\partial w_i} = F_i \frac{\partial R(\bar{S}, w_i)}{\partial w_i} - F_N \frac{\partial R(\bar{S}, w_N)}{\partial w_N} = 0, \quad i = 1, .., N-1,$$

$$\frac{\partial Q}{\partial \bar{S}} = \sum_{i=1}^{N} F_i \frac{\partial R}{\partial \bar{S}} - c = 0,$$

$$F_1 R(\bar{S}, w_1) + F_2 R(\bar{S}, 1 - w_1) - c\bar{S} - \frac{dX(n)}{dn} \leq 0.$$

Note that $\bar{S}^P$ and w_i^P are determined by the first N first-order conditions and do not depend on n or α. Furthermore, when $n = 1$ assumption 4 implies $\frac{w_i^P}{w_j^P} = \frac{S_i^\alpha}{S_j^\alpha}$, thus an increase in α should be met with a more equal distribution of skills. In the online appendix we show the same holds for an increase in n when $N = 2$.

Should the planner face a restriction on the amount of skills (e.g. $\bar{S} \leq \bar{\bar{S}}$), the first-order condition for w_i^P would be the same and the profit maximizing win probability would only change if $\bar{S}$ affects differently teams' marginal revenue from win probability.

To guarantee uniqueness of the planner's allocation, the objective must be strictly concave which requires a negative definite Hessian matrix. Since by assumption $\frac{d^2X(n)}{dn^2} > 0$, a sufficient condition for this is $\frac{\partial^2 R}{\partial \bar{S} \partial w_i} = 0$, which is satisfied under assumption 2. Under this assumption we furthermore have:

$$w_i^P = \frac{F_i}{\sum_{i=1}^N F_i},$$

$$\bar{S}^P = \left(\frac{\sigma(\sum_{i=1}^N F_i)}{c} \right)^{\frac{1}{1-\sigma}}.$$

6.2 Proof of Proposition 3

From the first order conditions for hiring talent,

$$\frac{\partial R}{\partial S_i} + \frac{\partial R}{\partial w_i} \frac{dw_i}{dw_{ij}} \frac{dw_{ij}}{dS_i} - \frac{c}{F_i} = 0.$$

An increase in α will affect the terms $\frac{dw_i}{dw_{ij}} \frac{dw_{ij}}{dS_i}$, while an increase in n will only affect the term $\frac{dw_i}{dw_{ij}}$. The lemma in the online appendix shows that if $w_{12} < \overline{w_{12}}(n)$, then an increase in n will lead to an increase in incentives to hire, i.e. it will shift the $S_i^*(S_j)$ reaction functions upwards. This will increase the Nash equilibrium hiring levels, S_1^* and S_2^*. To find the effects of an increase in α, we start from

$$\frac{dw_i}{dw_{ij}}\frac{dw_{ij}}{dS_i} = \alpha \frac{w_{ij}^{\frac{n+1}{2}}(1-w_{ij})^{\frac{n+1}{2}} n!}{S_i(\frac{n-1}{2}!)^2}.$$

Its derivative with respect to α is given by

$$\begin{aligned}
\frac{d\frac{dw_i}{dw_{ij}}\frac{dw_{ij}}{dS_i}}{d\alpha} &= \frac{w_{ij}^{\frac{n+1}{2}}(1-w_{ij})^{\frac{n+1}{2}} n!}{S_i(\frac{n-1}{2}!)^2} \\
&\quad + \alpha \frac{n!\frac{n+1}{2}}{S_i(\frac{n-1}{2}!)^2} w_{ij}^{\frac{n+1}{2}}(1-w_{ij})^{\frac{n+1}{2}} \left[(1-w_{ij})\ln S_i + w_{ij}\ln S_j\right], \\
&= \frac{dw_i}{dw_{ij}}\frac{dw_{ij}}{dS_i}\left(\frac{1}{\alpha} + \frac{n+1}{2}\left[(1-w_{ij})\ln S_i + w_{ij}\ln S_j\right]\right).
\end{aligned}$$

When $\alpha > 0$, this derivative might be negative only when one of the teams' skill choice is close to zero. A sufficient condition for the derivative to be positive would be that F_1 and F_2 are large enough such that $S_2 = 1$ (since then $S_1 > 1$). A lower threshold for α, $\underline{\alpha}(F_1, F_2)$, such that this sufficient condition is satisfied, is characterized by the following system of equations from teams' first order conditions

$$\begin{aligned}
\frac{\partial R}{\partial \bar{S}}|_{\bar{S}=S_1+1} + \underline{\alpha}(F_1, F_2)\frac{w_{12}^{\frac{n+1}{2}}(1-w_{12})^{\frac{n+1}{2}} n!}{S_1(\frac{n-1}{2}!)^2}\frac{\partial R}{\partial w}|_{w=w_1} &= \frac{c}{F_1}, \\
\frac{\partial R}{\partial \bar{S}}|_{\bar{S}=S_1+1} + \underline{\alpha}(F_1, F_2)\frac{w_{12}^{\frac{n+1}{2}}(1-w_{12})^{\frac{n+1}{2}} n!}{(\frac{n-1}{2}!)^2}\frac{\partial R}{\partial w}|_{w=1-w_1} &= \frac{c}{F_2}.
\end{aligned}$$

Since $\lim_{\bar{S}\to\infty}\frac{\partial R}{\partial \bar{S}} = 0$, it must be that $\lim_{F_2\to\infty}\underline{\alpha}(F_1, F_2) = 0$. Thus, increases in α when $\alpha > \underline{\alpha}(F_1, F_2)$ always increases the Nash equilibrium hiring levels, S_1^* and S_2^*.

6.3 Proof of proposition 5

First, we are going to show that when $\alpha > \bar{\alpha}(1)$, a pair $0 < \tau < 1$, and s exists such that both teams first order conditions are satisfied when evaluated at the optimal allocation. Using the planner's profit maximizing first order conditions, we can rewrite the marginal effect of hiring for team 1, evaluated at the profit maximizing allocation (and thus satisfying $(F_1 + F_2)\frac{\partial R}{\partial \bar{S}} = c)$, as

$$\begin{aligned}
& (1-\frac{\tau}{2})F_1\frac{dR}{dS_1} + \frac{\tau}{2}F_2\frac{dR}{dS_1} - (F_1+F_2)\frac{\partial R}{\partial \bar{S}} \\
= & -\frac{\tau}{2}F_1\frac{\partial R}{\partial S_1} - (1-\frac{\tau}{2})F_2\frac{\partial R}{\partial S_1} + (1-\frac{\tau}{2})F_1\frac{\partial R}{\partial w_1}\frac{\partial w_1}{\partial S_1} + \frac{\tau}{2}F_2\frac{\partial R}{\partial w_2}\frac{\partial w_2}{\partial S_1} \\
= & -\frac{\tau}{2}F_1\frac{\partial R}{\partial S_1} - (1-\frac{\tau}{2})F_2\frac{\partial R}{\partial S_1} + (1-\tau)F_1\frac{\partial R}{\partial w_1}\frac{\partial w_1}{\partial S_1}. \qquad (12)
\end{aligned}$$

Where in the last equality we have used $F_2\frac{\partial R}{\partial w_2} = F_1\frac{\partial R}{\partial w_1}$ when evaluated at the profit maximizing allocation, and $\frac{\partial w_2}{\partial S_1} = -\frac{\partial w_1}{\partial S_1}$.

When $\tau = 1$ the above equation is negative. And when $\tau = 0$ it is simply the first order condition for team 1 in the absence of regulation evaluated at $S_i = S_i^P$. If $\alpha > \bar{\alpha}(1)$, then teams in autarky would hire inefficiently high levels of talent, i.e. $S_1^* + S_2^* > \bar{S}^P$. In this case, the first order condition of at least one team would be positive when evaluated at $S_i = S_i^P$. We later provide assumptions for the first order condition of team 1 to be higher than that of team 2 when evaluated at the planner's allocation. For now we proceed under the assumption that the marginal benefit of hiring for team 1 when evaluated at $S_i = S_i^P$ is larger than that of team 2 when $\alpha > \bar{\alpha}(1)$.[48] Therefore, by continuity, when $\alpha > \bar{\alpha}(1)$ there exists a $0 < \tau < 1$ that would make team 1 to choose $S_1 = S_1^P$ when team 2 is choosing S_2^P.

Now we evaluate team 2's first order condition at the profit-maximizing allocation and the tax rate that makes team 1 to choose S_1^P. This gives

$$\begin{aligned}
& (1-\frac{\tau}{2})F_2\frac{dR}{dS_2} + \frac{\tau}{2}F_1\frac{dR}{dS_2} - (F_1+F_2)\frac{\partial R}{\partial \bar{S}} + cs, \\
= & -\frac{\tau}{2}F_2\frac{\partial R}{\partial S_2} - (1-\frac{\tau}{2})F_1\frac{\partial R}{\partial S_2} + (1-\tau)F_2\frac{\partial R}{\partial w_2}\frac{\partial w_2}{\partial S_2} + cs.
\end{aligned}$$

Clearly there always exists a subsidy/tax s that would make this equation to equal zero when $S_2 = S_2^P$. Now we want to prove that $s > 0$, i.e. the league subsidizes team 2. To do this we substract team 1's first order condition from team 2's to get

$$(1-\tau)\left[(F_2 - F_1)\frac{\partial R}{\partial \bar{S}} + F_2\frac{\partial R}{\partial w_2}\frac{\partial w_2}{\partial S_2} - F_1\frac{\partial R}{\partial w_1}\frac{\partial w_1}{\partial S_1}\right] + cs. \qquad (13)$$

[48]Otherwise, the proof would go along using a parallel reasoning, but it would require the strong team to be given a hiring subsidy.

If $\tau = s = 0$ (13) is negative, under our assumption that the first order condition of team 1 is higher than that of team 2 when evaluated at the planner's allocation. This implies that the term in brackets is negative, and thus that s must be positive if (13) is to be zero. Therefore, the weak team is subsidized. We conclude then that the profit maximizing allocation can be decentralized through a combination of revenue sharing and a hiring subsidy for the weak team provided that the autarky allocation satisfies $\alpha > \bar{\alpha}(1)$.

To prove that an increase in α leads to an increase in optimal level of revenue sharing we consider the effect that this has on (12). The only term that is directly affected is $\frac{\partial w_1}{\partial S_1}$, which we found increases with α when $\alpha > \underline{\alpha}(F_1, F_2)$. Thus to keep this first order condition at zero the revenue sharing tax, τ, has to increase as well: $\frac{d\tau}{d\alpha} > 0$.

The result that an increase in α reduces the dispersion of skills across teams follows from the fact that in the optimal allocation, w_1^P is independent of α but w_1 is decreasing in α for a given skills ratio, since $\frac{w_{12}}{w_{21}} = \frac{S_1^\alpha}{S_2^\alpha}$. Thus $\frac{S_1}{S_2}$ has to decrease with α.

Under assumption 2 we can use the implicit function theorem to perform other comparative statics analysis with respect to a reduction of k (which amounts to an increase in the relative importance of broadcasting revenue), a proportional increase in F_i, or an increase in the dispersion of F_i (which amounts to an expansion of the league's size, N). We find,

$$\frac{d\tau}{d(-k)} < 0, \qquad \frac{d\tau}{dF_i} < 0, \qquad \frac{d\tau}{d\frac{F_1}{F_2}} \lesseqgtr 0.$$

Thus, both a proportional increase in fan bases, or an increase in the relative importance of broadcasting revenue lead to a lower level of revenue sharing. Intuitively, this is due to both effects increasing the importance of quality-related revenue sources relative to revenues associated with winning the championship. This aligns teams' incentives to the league and less revenue sharing is required. Expanding the league has an ambiguous effect on revenue sharing (the lower the α the more likely it will be negative).

When $\alpha < \bar{\alpha}(1)$, an increase in season length, measured by n, will increase teams' incentives to hire skills if $w_{12} < \overline{w_{12}}(1)$, as shown in the online appendix. Since increasing n increases the league's fixed costs, $X(n)$, the league might choose not to implement the profit maximizing allocation, i.e. $S_1^* + S_2^* < \bar{S}^P$. As n is a discrete variable, if the league finds it optimal to choose a n^P such that $\alpha > \bar{\alpha}(n^P)$, then the league employs revenue sharing such that the profit-maximizing allocation, S_1^P, S_2^P is attained. If $\alpha < \bar{\alpha}(n^P)$, then $S_1^* + S_2^* < \bar{S}^P$ and proposition 5 is uninformative about other institution choices.

We now characterize conditions for the first order condition of team 1 to be higher than that of team 2 when evaluated at the planner's allocation. Under assumption 2, and for simplicity assuming $n = 1$, the difference in first order conditions at the planner's allocation is given by

$$\sigma\left(\frac{\sigma(F_1+F_2)}{c}\right)^{-1}(F_1-F_2) + k\left(\frac{\sigma(F_1+F_2)}{c}\right)^{\frac{\sigma}{1-\sigma}}\frac{F_1F_2}{F_1+F_2}\left(\frac{1}{S_1^P}-\frac{1}{S_2^P}\right).$$

Replacing the planner's solution for (S_1^P, S_2^P) gives the following parametric condition for this difference to be positive,[49]

$$\sigma(F_1-F_2) > k\frac{F_1F_2}{F_1+F_2}\left(F_1^{\frac{1}{\alpha}}+F_2^{\frac{1}{\alpha}}\right)\left(\frac{1}{F_2^{\frac{1}{\alpha}}}-\frac{1}{F_1^{\frac{1}{\alpha}}}\right).$$

Where we dropped the term $\left(\frac{\sigma(F_1+F_2)}{c}\right)^{-1}$, which is positive and was multiplying both sides of the inequality. Imposing $F_1 = 2F_2$ we finally get

$$k < 3\sigma\frac{2^{\frac{1-\alpha}{\alpha}}}{2^{\frac{2}{\alpha}}-1}.$$

For given σ and α, there always exists k low enough to satisfy this condition.

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Properties of the probability of winning the championship, $W(\cdot)$

If the championship is determined by the outcome of n games (with n odd), given assumption 4 on the win probability of a game, then the probability that team 1 defeats team 2, $w_1 = W(S_1, S_2, \alpha, n)$, is given by

$$w_1 = \sum_{k=\frac{n+1}{2}}^{n}\binom{n}{k}w_{12}^k(1-w_{12})^{n-k}.$$

[49]Using $w_1 = \frac{S_1^\alpha}{S_1^\alpha+S_2^\alpha}$, and the planner's allocation, $S_i^P = \frac{F_i^{\frac{1}{\alpha}}}{F_1^{\frac{1}{\alpha}}+F_2^{\frac{1}{\alpha}}}\left(\frac{(F_1+F_2)\sigma}{c}\right)^{\frac{1}{1-\sigma}}$.

We want to determine the effect of an increase in n on teams' incentives to hire skills. We start by noting that the cumulative distribution function for the binomial distribution is given by the regularized incomplete beta function, i.e.[50]

$$H(k; n, w_{12}) = I_{1-w_{12}}(n-k, k+1) = 1 - I_{w_{12}}(k+1, n-k),$$

and that w_1 is then given by

$$w_1 = I_{w_{12}}(\frac{n+1}{2}, \frac{n+1}{2}).$$

The effect of an increase from n to $n+2$ on w_1 is given by

$$I_{w_{12}}(\frac{n+3}{2}, \frac{n+3}{2}) - I_{w_{12}}(\frac{n+1}{2}, \frac{n+1}{2}) \quad = \quad \left[I_{w_{12}}(\frac{n+1}{2}, \frac{n+3}{2}) - \frac{w_{12}^{\frac{n+1}{2}}(1-w_{12})^{\frac{n+3}{2}}}{\frac{n+1}{2} B(\frac{n+1}{2}, \frac{n+3}{2})} \right]$$

$$- \left[I_{w_{12}}(\frac{n+1}{2}, \frac{n+3}{2}) - \frac{w_{12}^{\frac{n+1}{2}}(1-w_{12})^{\frac{n+1}{2}}}{\frac{n+1}{2} B(\frac{n+1}{2}, \frac{n+1}{2})} \right] \quad = \quad \frac{w_{12}^{\frac{n+1}{2}}(1-w_{12})^{\frac{n+1}{2}} n!}{\frac{n+1}{2}\frac{n-1}{2}!\frac{n-1}{2}!} [1 - 2(1-w_{12})] > 0.$$

Where properties of the regularized beta function were used, and $B(x, y)$ is the beta function, which when x and y are positive integers is given by $B(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$. The last inequality follows since $1 - w_{12} < \frac{1}{2}$.

When making hiring decisions, teams estimate

$$\frac{\partial R}{\partial w_i} \frac{dw_i}{dw_{ij}} \frac{dw_{ij}}{dS_i}.$$

Thus an increase in n will affect incentives through its effect on the term $f(n) \equiv \frac{dw_i}{dw_{ij}}$. When $f(n) > f(n-2)$ (with $f(1) = 1$), team i will have higher incentives to hire talent with an increase in season length. From the expression for the regularized beta function we can derive

$$f(n) = \frac{w_{12}^{\frac{n-1}{2}}(1-w_{12})^{\frac{n-1}{2}} n!}{\frac{n-1}{2}!\frac{n-1}{2}!}.$$

To determine conditions for an increase in season length to increase teams' incentive to hire talent we are going to look at the ratio $\frac{f(n+2)}{f(n)}$:

$$\frac{f(n+2)}{f(n)} = \frac{(n+2)(n+1)}{\left(\frac{n+1}{2}\right)^2} w_{12}(1-w_{12}).$$

[50] Here H is the cumulative distribution function for observing k or less wins for team 1 when the probability of one win is w_{12} and n games are played, and I is the regularized beta function.

The terms related to n are greater than one, but decreasing with n. But this ratio is not always larger than 1, since this will be violated when w_{12} is close to 1. For an increase of season length to boost hiring incentives it must be the case that

$$w_{12} < \overline{w_{12}}(n) \equiv \frac{1 + \sqrt{\frac{1}{n+2}}}{2}$$

For example, when $n = 1$

$$\overline{w_{12}}(1) \approx 0.788.$$

Consider the case of the profit maximizing allocation, relevant according to proposition 5, under assumption 2. In 6.1 we found (since $F_1 = 2F_2$) that $w_{12}^P = \frac{2}{3}$. Thus an increase in season length would increase hiring incentives (we note that this would be the case as long as $\frac{F_1}{F_2} \lesssim 3.72$).

Lemma

Under assumption 2, and when $n = 1$, teams' revenue functions are strictly concave in own skill: $\frac{d^2R(\bar{S},w_i)}{dS_i^2} < 0$. When $n > 1$, team 1's revenue function is strictly concave in own skills.

Proof of Lemma: For team i, the derivative of the revenue function is:

$$\begin{aligned}\frac{dR}{dS_i} &= \frac{\partial R}{\partial \bar{S}} + \frac{\partial R}{\partial w_i}\frac{dw_i}{dS_i} = \sigma \bar{S}^{\sigma-1} + K\frac{1}{w_i}\frac{dw_i}{dS_i} \\ &= \sigma \bar{S}^{\sigma-1} + K\frac{1}{w_i}\frac{dw_i}{dw_{ij}}\frac{\alpha w_{ij}(1-w_{ij})}{S_i} \\ &= \sigma \bar{S}^{\sigma-1} + K\frac{1}{w_i}\frac{w_{ij}^{\frac{n-1}{2}}(1-w_{ij})^{\frac{n-1}{2}}n!}{\frac{n-1}{2}!\frac{n-1}{2}!}\frac{\alpha w_{ij}(1-w_{ij})}{S_i} \\ &= \sigma \bar{S}^{\sigma-1} + K\frac{1}{w_i}\frac{\alpha w_{ij}^{\frac{n+1}{2}}(1-w_{ij})^{\frac{n+1}{2}}n!}{S_i\frac{n-1}{2}!\frac{n-1}{2}!}\end{aligned}$$

The second derivative is thus:

$$\begin{aligned}\frac{d^2R}{dS_i^2} &= \frac{\partial^2 R}{\partial \bar{S}^2} + \frac{\partial^2 R}{\partial w_i^2}\left(\frac{dw_i}{dS_i}\right)^2 + \frac{\partial R}{\partial w_i}\frac{d^2 w_i}{dS_i^2} \\ &= \frac{\partial^2 R}{\partial \bar{S}^2} + \frac{\partial^2 R}{\partial w_i^2}\left(\frac{dw_i}{dS_i}\right)^2 + \frac{\partial R}{\partial w_i}\frac{\alpha n! w_{ij}^{\frac{n+1}{2}}(1-w_{ij})^{\frac{n+1}{2}}\left[\left(\frac{n+1}{2}\right)(1-2w_{ij})\alpha - 1\right]}{\frac{n-1}{2}!\frac{n-1}{2}!S_i^2}\end{aligned}$$

Since for team 1, $w_{12} > \frac{1}{2}$, the second derivative is always negative when evaluated at S_1^*, S_2^*. For team 2 a sufficient condition for it to be negative is that $n = 1$ (this requires combining the second and third terms in the above equation). For $n > 1$, a sufficient condition for the objective function for team 2 to be concave is

$$\left(\frac{n+1}{2}\right)(1-2w_{21})\alpha - 1 - \frac{\alpha n! w_{21}^{\frac{n+1}{2}}(1-w_{21})^{\frac{n+1}{2}}}{w_2 \frac{n-1}{2}! \frac{n-1}{2}!} < 0.$$

This gives an implicit restriction on parameters. Note that since $w_{21} = 0$ does not correspond to a minimum of the last term, this restriction is looser than $\alpha < \frac{2}{n+1}$. □

Proof of proposition 2

Uniqueness of teams' solutions to their first order conditions follows directly under parameter restrictions from the Lemma above that guarantee concavity of the objective functions. If we plot the graphs of $S_1^*(S_1), S_2^*(S_1)$ against each other in (S_1, S_2)-space, Nash-equilibria of the model occur at (and only at) intersections of the two graphs. Based on this, we prove equilibrium existence in two steps:

i. By showing that $\lim_{S_j \to 0} S_i^*(S_j) = \left(\frac{\sigma F_i}{c}\right)^{\frac{1}{1-\sigma}} > 0$.

ii. By showing that $\lim_{S_j \to \infty} S_i^*(S_j) = \frac{K\alpha \frac{n+1}{2} F_i}{c} < S_j$.

We now go through the two steps. Step 1: From the first order condition for team i, (1), when $S_j \to 0$, choice of S_i is given by

$$\sigma S_i^{\sigma-1} + K\frac{1}{w_i}\frac{\alpha w_{ij}^{\frac{n+1}{2}}(1-w_{ij})^{\frac{n+1}{2}} n!}{S_i \frac{n-1}{2}! \frac{n-1}{2}!} - \frac{c}{F_i} \leq 0.$$

Note that if $S_i = 0$ then $w_i = 0$ and $w_{ij} = 0$. Using L'Hôpital's rule we find that the first two terms diverge in this case. Thus $S_i > 0$ which implies $w_i = w_{ij} = 1$ and S_i given by

$$S_i = \left(\frac{\sigma F_i}{c}\right)^{\frac{1}{1-\sigma}}.$$

Step 2: From the first order condition for team i, (1), when $S_j \to \infty$, choice of S_i is given by

$$K\frac{1}{w_i}\frac{\alpha w_{ij}^{\frac{n+1}{2}}(1-w_{ij})^{\frac{n+1}{2}} n!}{S_i \frac{n-1}{2}! \frac{n-1}{2}!} - \frac{c}{F_i} \leq 0.$$

First we note that $w_i = 0$, since if $w_i > 0$, the first order condition would imply $S_i < \infty$, which would be a contradiction. Next to derive S_i we apply L'Hôpital's rule. This gives

$$S_i = \frac{K\alpha^{\frac{n+1}{2}} F_i}{c}.$$

Since the best responses $S_i^*(S_j)$ are continuous, the above implies that $S_1^*(S_2)$ and $S_2^*(S_1)$ must cross at least once and thus that there is at least one Nash equilibrium in the model.

To prove uniqueness, with no loss of generality let's assume that S_1^* and S_2^* is the equilibrium with lowest total skills, i.e. if there exists another equilibrium, S_1', S_2', it has to be that $S_1' + S_2' > S_1^* + S_2^*$ (that there cannot exist two distinct equilibria with same total skills is straightforward from (1)).

Consider the following expression that is derived from the difference of teams' first order conditions at both equilibria

$$\left[{w_{12}^*}^{\frac{n+1}{2}} (1 - w_{12}^*)^{\frac{n+1}{2}} \right] \left(\frac{1}{w_1^*} - \frac{S_1^*}{S_2^*} \frac{1}{w_2^*} \right) \frac{1}{S_1^*} = \left[{w_{12}'}^{\frac{n+1}{2}} (1 - w_{12})'^{\frac{n+1}{2}} \right] \left(\frac{1}{w_1'} - \frac{S_1'}{S_2'} \frac{1}{w_2'} \right) \frac{1}{S_1'}.$$

Let's assume first that $\frac{S_1'}{S_2'} \geq \frac{S_1^*}{S_2^*}$. This implies that $w_1' \geq w_1^*$, $S_1' > S_1^*$, and that the term in square brackets is lower in the new equilibrium (since $w_{12}' > w_{12}^* > \frac{1}{2}$). Thus the right hand side would be lower than the left hand side and S_1', S_2' cannot be an equilibrium. A similar reasoning rules out equilibria with $\frac{S_1'}{S_2'} < \frac{S_1^*}{S_2^*}$ and $S_1' < S_1^*$ (the right hand side would be higher than the left hand side and thus the equality cannot hold).

To rule out the remaining case of $\frac{S_1'}{S_2'} < \frac{S_1^*}{S_2^*}$ and $S_1' > S_1^*$, start with the first order condition for team 2 at the initial equilibrium: $\frac{dR}{dS_2} = \frac{\partial R}{\partial S_2} + \frac{\partial R}{\partial w_2} \frac{\partial w_2}{\partial S_2} = \frac{c}{F_2}$. Since the objective function is concave, then we know that increasing S_2 to S_2' (keeping S_1 constant) will lead to a reduction of the terms in the first order condition. More importantly, under assumption 1 all three terms in the first order condition are lower at this point, which we denote as "interim 1". Now. let's increase S_1 up to the candidate equilibrium, i.e. a point where $\frac{S_1'}{S_2'}$ is lower than initial equilibrium, and also up to the point where $\frac{S_1}{S_2}$ is the same as in the initial equilibrium, we call the latter the "interim 2" point. At the "interim 2" point, $\frac{\partial w_2}{\partial S_2} = \alpha \frac{w_{21}^{\frac{n+1}{2}} (1-w_{21})^{\frac{n+1}{2}} n!}{S_2' \frac{n-1}{2}! \frac{n-1}{2}!}$ is stricly lower since $S_2' > S_2^*$. Now since at the candidate equilibrium $\frac{S_1'}{S_2'} < \frac{S_1^*}{S_2^*}$, it must be the case that $\frac{\partial R}{\partial w_2}$ is lower than in the initial equilibrium (since w_2 is higher), and $\frac{\partial w_2}{\partial S_2}$ has to be between its values at "interim 1" and "interim 2" (since S_2 is constant and only w_{12} changes, but always above $\frac{1}{2}$). Since in both counterfactual interim points, $\frac{\partial w_2}{\partial S_2}$ is below its value in the initial equilibrium, then in the candidate equilibrium this must also be the case. Finally, in the candidate equilibrium $S_1' + S_2' > S_1^* + S_2^*$, therefore $\frac{\partial R}{\partial S_2}$ is below its value in the initial equilibrium. Thus, we conclude that in the candidate equilibrium $\frac{\partial R}{\partial S_2'} + \frac{\partial R}{\partial w_2'}\frac{\partial w_2'}{\partial S_2'} < \frac{c}{F_2}$.

This proves that no equilibrium with $S_1' + S_2' > S_1^* + S_2^*$ exists, and thus that the model has a unique interior Nash equilibrium.

Proof of Proposition 4

Assumption 2 implies

$$\left(1-\frac{\tau}{2}\right) F_i \frac{d^2 R}{dS_i^2} + \frac{\tau}{2} F_j \left(\frac{\partial^2 R}{\partial \bar{S}^2} + \frac{\partial^2 R}{\partial w_j^2}\left(\frac{dw_j}{dS_i}\right)^2 + \frac{\partial R}{\partial w_j}\frac{d^2 w_j}{dS_i^2}\right) < 0,$$

when evaluated at (S_1^*, S_2^*). Thus, teams' optimization problems still have a unique solution.

In parallel as we did in the proof of proposition 2, and assuming for simplicity $n = 1$, we prove equilibrium existence by studying properties of best response functions $S_1^*(S_1), S_2^*(S_1)$ and showing:

i. By showing that $\lim_{S_j \to 0} S_i^*(S_j) > 0$.

ii. By showing that $\lim_{S_j \to \infty} S_i^*(S_j) < \infty$.

We now go through the two steps. Step 1: From the first order condition for team i, (2), when $S_j \to 0$, choice of S_i is given by

$$\sigma S_i^{\sigma-1}\left((1-\frac{\tau}{2})F_i + \frac{\tau}{2}F_j\right) + \frac{\alpha K}{S_i}\left((1-\frac{\tau}{2})F_i(1-w_i) - \frac{\tau}{2}F_j w_i\right) - c(1 - s\mathbb{1}_{i=2}) \leq 0.$$

Note that if $S_i = 0$ then $w_i = 0$ and the first two terms diverge. Thus $S_i > 0$ which implies $w_i = 1$ and S_i characterized by

$$\sigma S_i^{\sigma-1}\left((1-\frac{\tau}{2})F_i + \frac{\tau}{2}F_j\right) - \frac{\alpha K}{S_i}\left(\frac{\tau}{2}F_j\right) - c(1 - s\mathbb{1}_{i=2}) = 0.$$

For every $0 < \tau \leq 1$ and $s < 1$, this equation has a unique solution $S_i > 0$.

Step 2: From the first order condition for team i, (2), when $S_j \to \infty$, choice of S_i is given by

$$\frac{\alpha K}{S_i}\left((1-\frac{\tau}{2})F_i(1-w_i) - \frac{\tau}{2}F_j w_i\right) - c(1 - s\mathbb{1}_{i=2}) \leq 0.$$

Note that if $S_i \to \infty$ this would be negative. This implies $S_i < \infty$, $w_i = 0$, and S_i given by

$$S_i = \frac{K\alpha(1-\frac{\tau}{2})F_i}{c(1-s\mathbb{1}_{i=2})} < \infty.$$

Since the best responses $S_i^*(S_j)$ are continuous, the above implies that $S_1^*(S_2)$ and $S_2^*(S_1)$ must cross at least once and thus that there is at least one Nash equilibrium in the model.

We now prove uniqueness. We start by assuming that S_1^* and S_2^* is the equilibrium with lowest total skills, i.e. if there exists another equilibrium, S_1', S_2', it has to be that $S_1' + S_2' > S_1^* + S_2^*$ (that there cannot exist two distinct equilibria with same total skills is straightforward from (2)).

Consider first the case that $\frac{S_1'}{S_2'} \geq \frac{S_1^*}{S_2^*}$. The FOC for team 1 can be rewritten as

$$\left(\left(1-\frac{\tau}{2}\right)F_1 + \frac{\tau}{2}F_2\right)\sigma(S_1'+S_2')^{\sigma-1} + K\frac{\alpha}{S_1'}\left(\left(1-\frac{\tau}{2}\right)F_1(1-w_1') - \frac{\tau}{2}F_2 w_1'\right) - c.$$

Since $S_1' + S_2' > S_1^* + S_2^*$, $S_1' \geq S_1^*$, and $w_1' \geq w_1^*$, the FOC for team 1 would be negative and we cannot have an equilibrium with $\frac{S_1'}{S_2'} \geq \frac{S_1^*}{S_2^*}$.

Consider now the case that $\frac{S_1'}{S_2'} < \frac{S_1^*}{S_2^*}$. The FOC for team 2 can be rewritten as

$$\left(\left(1-\frac{\tau}{2}\right)F_1 + \frac{\tau}{2}F_2\right)\sigma(S_1'+S_2')^{\sigma-1} + K\frac{\alpha}{S_2'}\left(-\left(1-\frac{\tau}{2}\right)F_1(1-w_1') + \frac{\tau}{2}F_2 w_1'\right) - c(1-s).$$

Since $S_1' + S_2' > S_1^* + S_2^*$, $S_2' > S_2^*$, and $w_1' < w_1^*$, the FOC for team 2 would be negative and we cannot have an equilibrium with $\frac{S_1'}{S_2'} < \frac{S_1^*}{S_2^*}$.

This proves that no equilibrium with $S_1' + S_2' > S_1^* + S_2^*$ exists, and thus that the model has a unique interior Nash equilibrium.